Multiresolution Decomposition and Visualization of 3D Scalar and Tensor Fields

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Scalar and tensor fields

Scientific analysis and visualization frequently implies the use of fields of some mathematical notion. Most acquisition devices provide scalars, vectors and tensor fields as a result of some high volume imaging process:

- remote sensing and photometry (topography and images)
- Medical scanners
  - magnetic resonance images (scalar densities)
  - diffusion tensor magnetic resonance (tensors)
  - ultrasound devices
- radio-astronomy telescope
- quantum-mechanical phenomena
- spin simulation of magnetic materials (vectors)
- long time varying processes
Problems

- Samples have complex interconnections with their neighbors and cannot be analysed individually: large subsets and/or the whole field must be viewed/analysed at once.

- Difficulties
  - Data known only on discrete sets (sampled data)
  - Complex data (non-negative definite matrices, and vectors): carry multivariate data which is harder to represent graphically.
  - Measurements are averaged (Partial Volume Effects PVA)
  - Noise
  - High dimensionality of data ($10^7$ values: high computational complexity)
  - perspective projection
Classical approaches

- cross sections (usually planar)
- geometric tensor representations (superquadric glyphs)
- isolines and isosurfaces extraction (fiber tracking)
- volume rendering (ray-casting methods)
- composition effects (color coding, transparency, illumination models)
- dynamic particle systems

We need to use methods from computer graphics, image processing, computer vision, and other areas to visually display, enhance, and manipulate information to allow better understanding of the data.

Zhukov and Barr, 2003
Our approach

- In our work, we combine multiresolution decomposition, orientation tensors and dynamic objects to view 3D scalar and tensor fields:
  - multiresolution decompositions can help volume field viewing
  - orientation tensors are suitable for capturing multivariate information
  - dynamic elements attract perceptual attention
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• The first results of this research line are
  ◦ multiresolution edge detection (ICCS’09 to appear, ICIP’09 submitted)
  ◦ 3D tensor field visualization with multiresolution decomposition
Multiresolution edge detection


- The image is decomposed into several scales using the Discrete Wavelet Transform (DWT).
- The resulting detail spaces form vectors indicating intensity variations which are combined using orientation tensors.

A high frequency scalar descriptor is then obtained from the resulting tensor for each original image pixel.
Wavelet transform

Decomposes signals over dilated and translated wavelets. A wavelet is a function $\psi \in L^2(\mathbb{R})$ with a zero average:

$$\int_{-\infty}^{+\infty} \psi(t) dt = 0$$

It is normalized $||\psi|| = 1$, and centered in the neighborhood of $t = 0$. A family of time-frequency atoms is obtained by scaling $\psi$ by $s$ and translating it by $u$:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}} \psi \left( \frac{t - u}{s} \right)$$

We are interested in wavelets which form a base of $L^2(\mathbb{R}^2)$ to represent images. If we have an orthonormal wavelet basis in $L^2(\mathbb{R})$ given by $\psi$ with the scaling function $\phi$, we can use

$$\psi^1(x) = \phi(x_1)\psi(x_2), \ \psi^2(x) = \psi(x_1)\phi(x_2), \ \psi^3(x) = \psi(x_1)\psi(x_2)$$
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<table>
<thead>
<tr>
<th>Row</th>
<th>Column</th>
<th>Diagonal</th>
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<tbody>
<tr>
<td>$\psi^1(x) = \phi(x_1)\psi(x_2)$</td>
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</tbody>
</table>
High frequency vector

We define a vector \( v_{j,p} \in \mathbb{R}^3 \) given by the inner product

\[
v_{j,p} = [I \cdot \psi_{j,p}^1, I \cdot \psi_{j,p}^2, I \cdot \psi_{j,p}^3]^T
\]

at scale \( j \) and position \( p \in I \), where \( I \) is the input image.

Now we have detail vectors (high frequencies) for every pixel in every scale.
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The question is: how can we combine these vectors for every image pixel?
Orientation tensor

An orientation tensor $T$ is a symmetric semi-definite positive matrix. Thus, it can be decomposed using the Spectral Theorem. In $\mathbb{R}^3$, an orientation tensor $T$ can be represented by a spear (its main orientation), a plate and a ball

$$T = (\lambda_1 - \lambda_2)e_1e_1^T + (\lambda_2 - \lambda_3)e_2e_2^T + \lambda_3(e_3e_3^T)$$

where $\lambda_i$ are the eigenvalues corresponding to each eigenvector $e_i$.

In a tensor summation, $T = A + B$, the degrees of colinearity and coplanarity of main directions are captured.

\[\text{Spear} + \text{Plate} + \text{Ball}\]
Coding high frequency vectors as tensors

Code vectors as spear tensors:

\[ M_{j,p} = v_{j,p}v_{j,p}^T \]
Coding high frequency vectors as tensors

Code vectors as spear tensors:

Compute one tensor for every pixel at every scale:
Coding high frequency vectors as tensors

Code vectors as spear tensors:

\[
M_{j,p} = v_{j,p}v_{j,p}^T
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Compute one tensor for every pixel at every scale:

We find the final tensor \( M_{0,p} \) for each pixel of the original image using

\[
M_{0,p} = \sum_{j=1}^{n_j} k_j M_{j,p}
\]
Proposed method

The weight \( k_j \in \mathbb{R} \) assigned to each scale can be given by

\[
k_j = \frac{\sum_{n=1}^{n_p} \text{Trace}(M_{j,n})}{\sum_{k=1}^{n_j} \sum_{n=1}^{n_p} \text{Trace}(M_{k,n})},
\]

where \( n_p \) is the number of pixels.
Results

Using Daubechies1

1 scale

3 scales
Results

Using Daubechies3

1 scale

3 scales
Results

Using Daubechies1

1 scale

3 scales
Results

Using Daubechies3

1 scale

3 scales
Conclusions

- A method for high frequency assessment and visualization was proposed:
  - coincident frequencies in space domain are highlighted
  - useful for finding edges, textures, collinear structures and salient regions for computer vision methods
  - one may infer texture regions by tuning the number of scales
  - the discrete wavelet transform and the tensor summation can be easily parallelized

- The $\lambda_1 - \lambda_2$ scalar field is one of the most used orientation alignment descriptors
  - but other relations can be extracted from final pixel tensors
  - there is promising information coded in the tensor eigenvectors that should be investigated in future works

- the effect of the combination of several scales should be investigated

- the potential and limits of different wavelets should be clearly stated
3D tensor field multiresolution visualization

- The first goal was to study how the decomposition of orientation tensors can be exploited.
  - Tensors can be naturally decomposed
  - Multiresolution tensors are suitable to speed up some processes
  - What tensor detail space means?
- Now, our research is focused on the combination of multiresolution tensors with dynamic particles for tensor field visualization.
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References


- www.gcg.ufjf.br

Thank you!